Week 10 - Wednesday

COMP 2100

Last time

- What did we talk about last time?
- Finished Dijkstra's algorithm
- Bipartite graphs
- Started matching

Questions?

Project 3

Assignment 5

Matching

Bipartite graphs

- A bipartite graph is one whose nodes can be divided into two disjoint sets X and Y
- There can be edges between set X and set Y
- There are no edges inside set X or set Y
- A graph is bipartite if and only if it contains no odd cycles

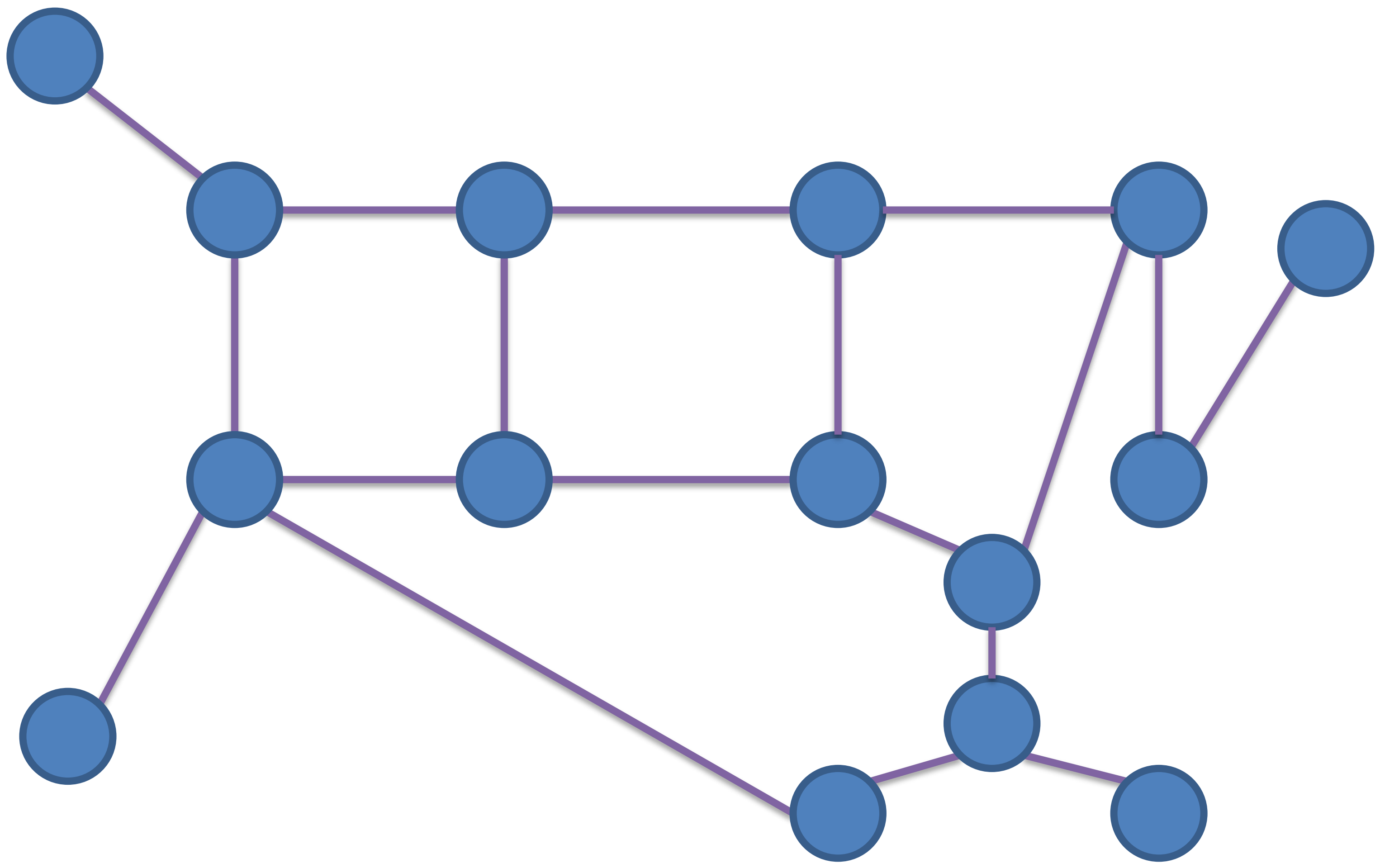
Maximum matching

- A **perfect matching** is when every node in set X and every node in set Y is matched
- It is not always possible to have a perfect matching
- We can still try to find a **maximum matching** in which as many nodes are matched up as possible

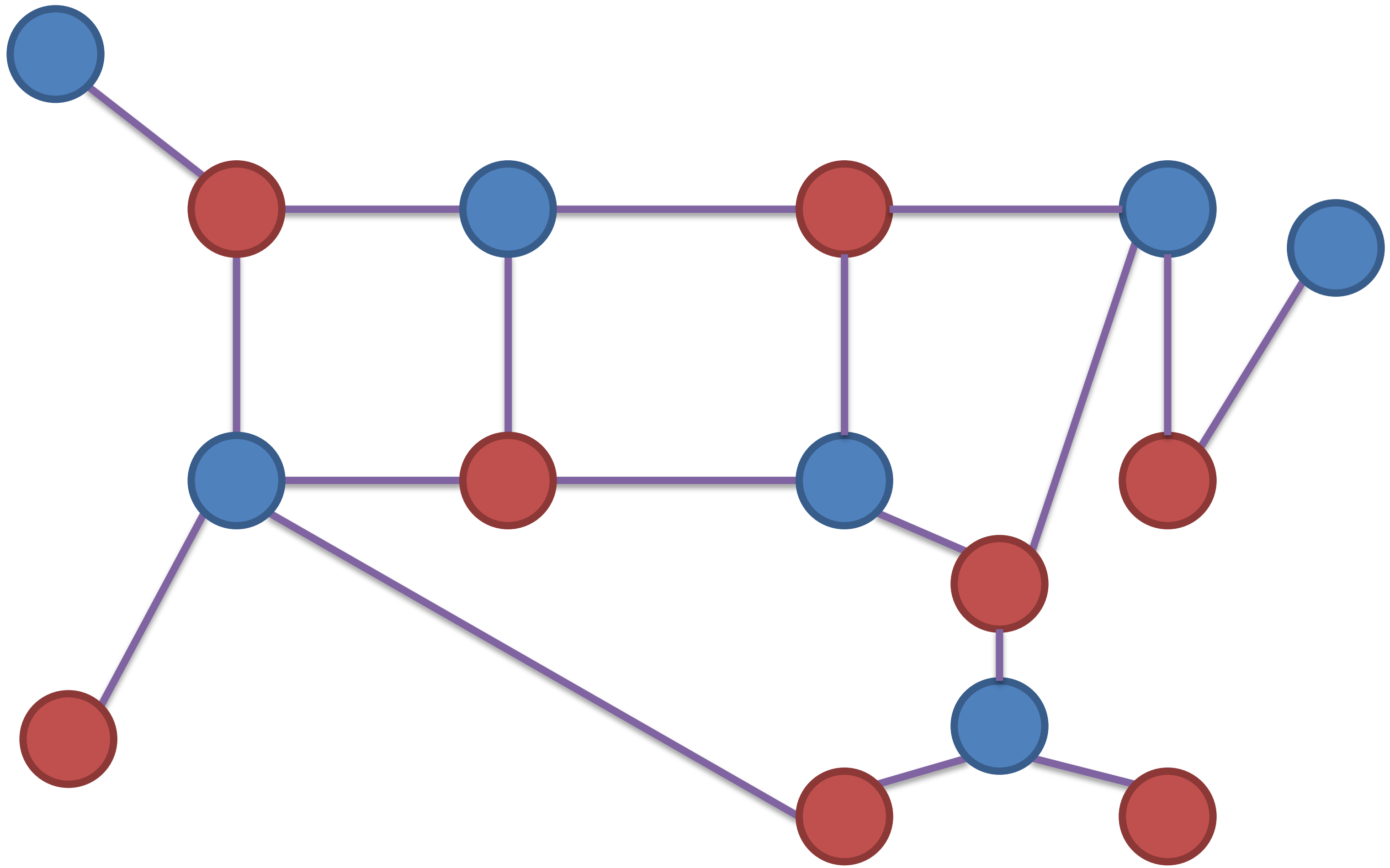
Matching algorithm

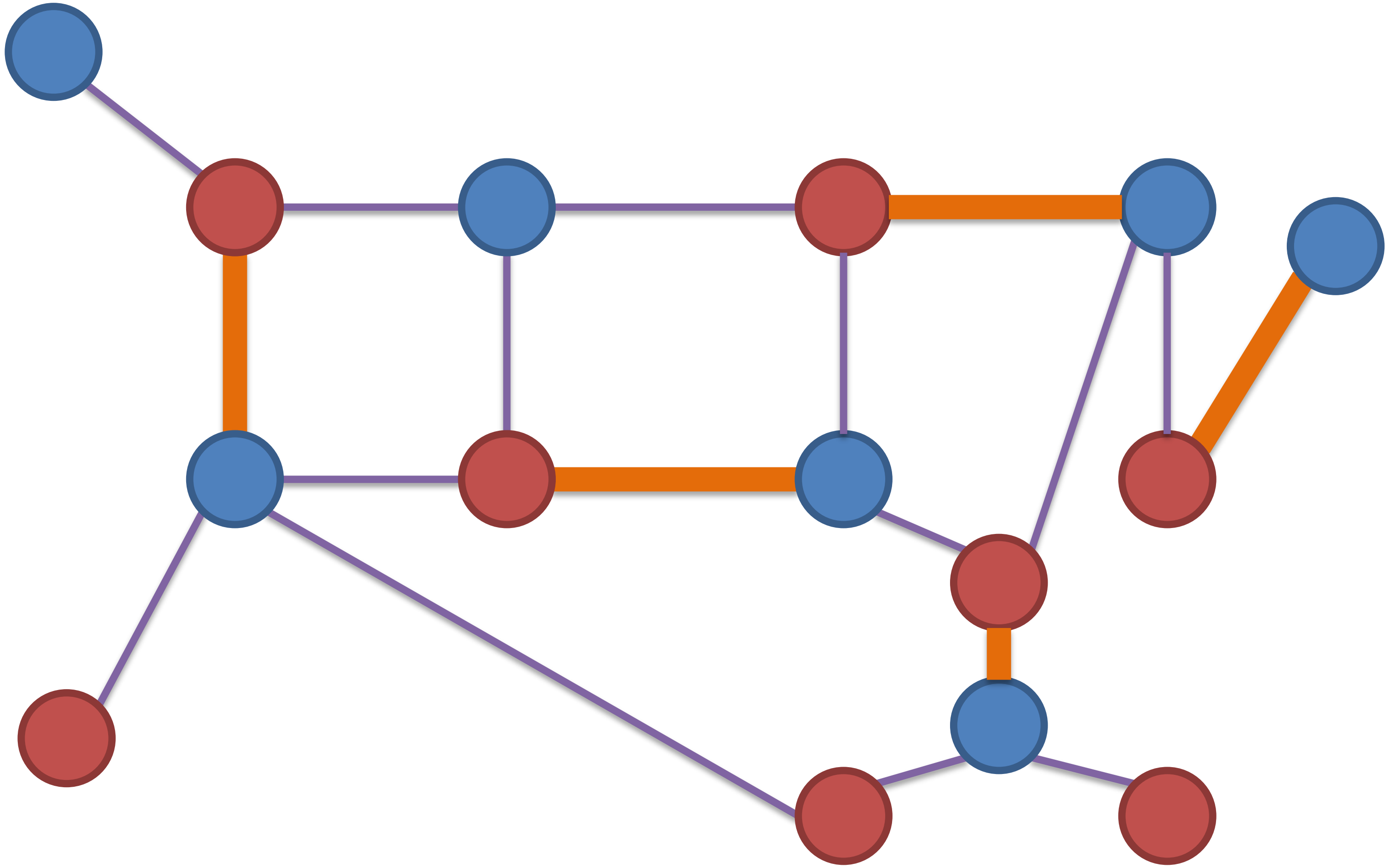
1. Come up with a legal, maximal matching
2. Take an **augmenting path** that starts at an unmatched node in X and ends at an unmatched node in Y, alternating the kind of edges it cross (first unmatched, then matched, then unmatched, etc.)
3. If there is such a path, switch all the edges along the path from being in the matching to being out and vice versa
4. If there's another augmenting path, go back to Step 2

Is this graph bipartite?

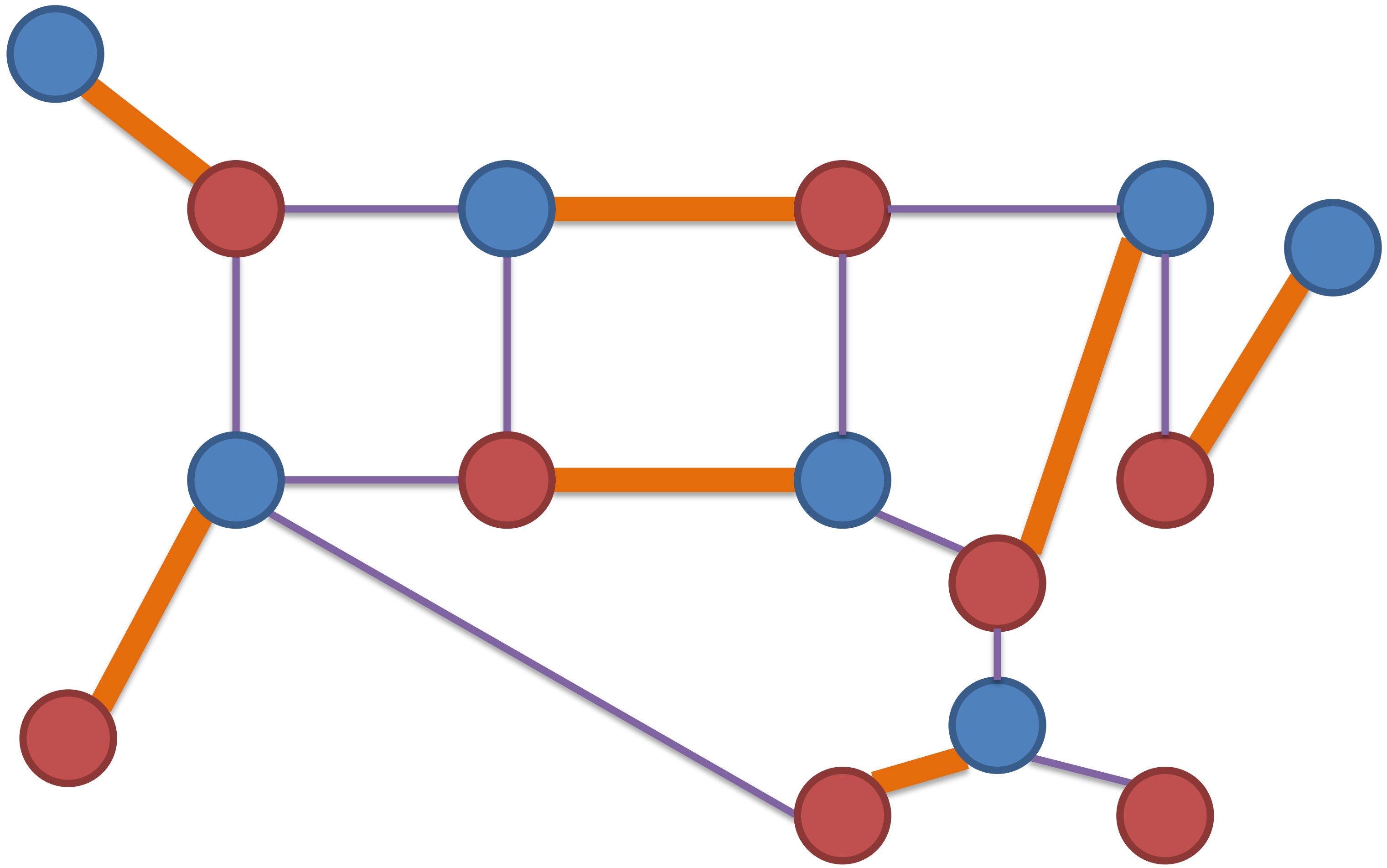


Let's make a maximal matching





Maximum matching! (one of them)



Stable Marriage

Imagine *n* men and *n* women

- All 2***n*** people want to get married
- Each woman has ranked all ***n*** men in order of preference
- Each man has ranked all ***n*** women in order of preference
- We want to match them up so that the marriages are **stable**

Stability

- Consider two marriages:
  - Anna and Bob
  - Caitlin and Dan
- This pair of marriages is unstable if
  - Anna likes Dan more than Bob **and** Dan likes Anna more than Caitlin **or**
  - Caitlin likes Bob more than Dan **and** Bob likes Caitlin more than Anna
- We want to arrange all ***n*** marriages such that none are unstable

Gale-Shapley algorithm

- ***n*** rounds
- In each round, every unengaged man proposes to the woman he likes best (who he hasn't proposed to already)
- An unengaged woman must accept the proposal from the one of her suitors she likes best
- If a woman is already engaged, she accepts the best suitor only if she likes him better than her fiancé

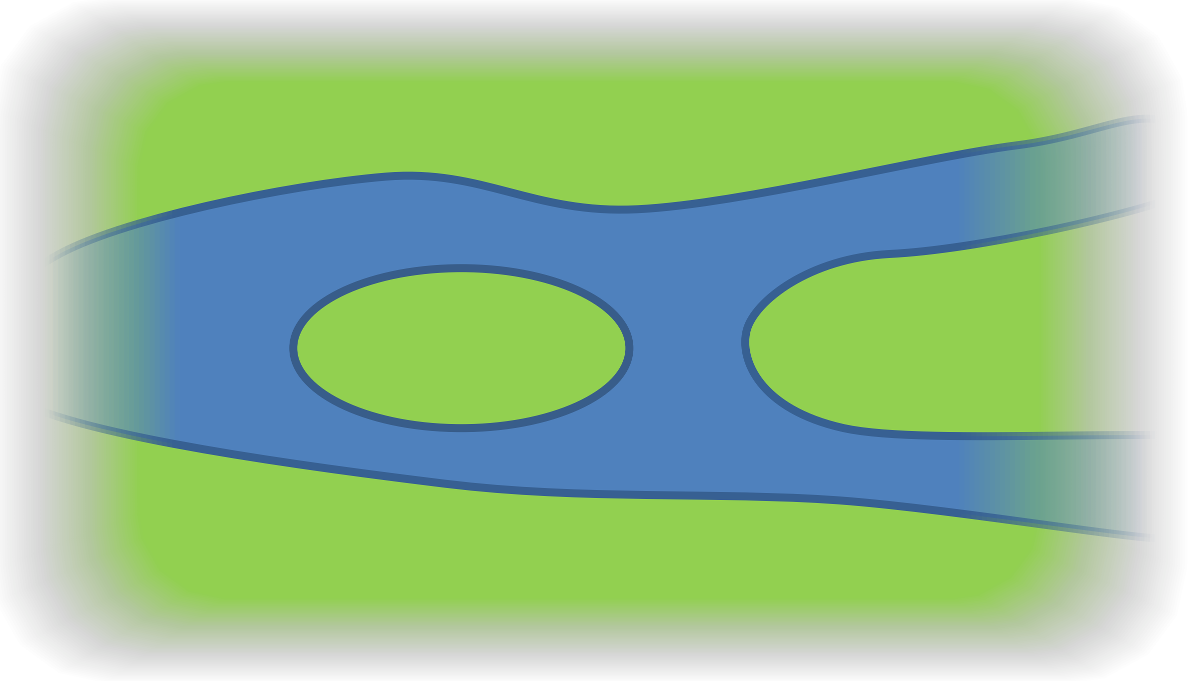
Lessons from Stable Marriage

- It's a graph problem where we never look at a graph
- The algorithm is guaranteed to terminate
  - A woman can't refuse to get engaged
  - If a man gets dumped, he will eventually propose to everyone
- The algorithm is guaranteed to find stable marriages
- Unfortunately, it's male-optimal and female-pessimal
- The lesson: Women should propose to men

Euler Paths and Tours

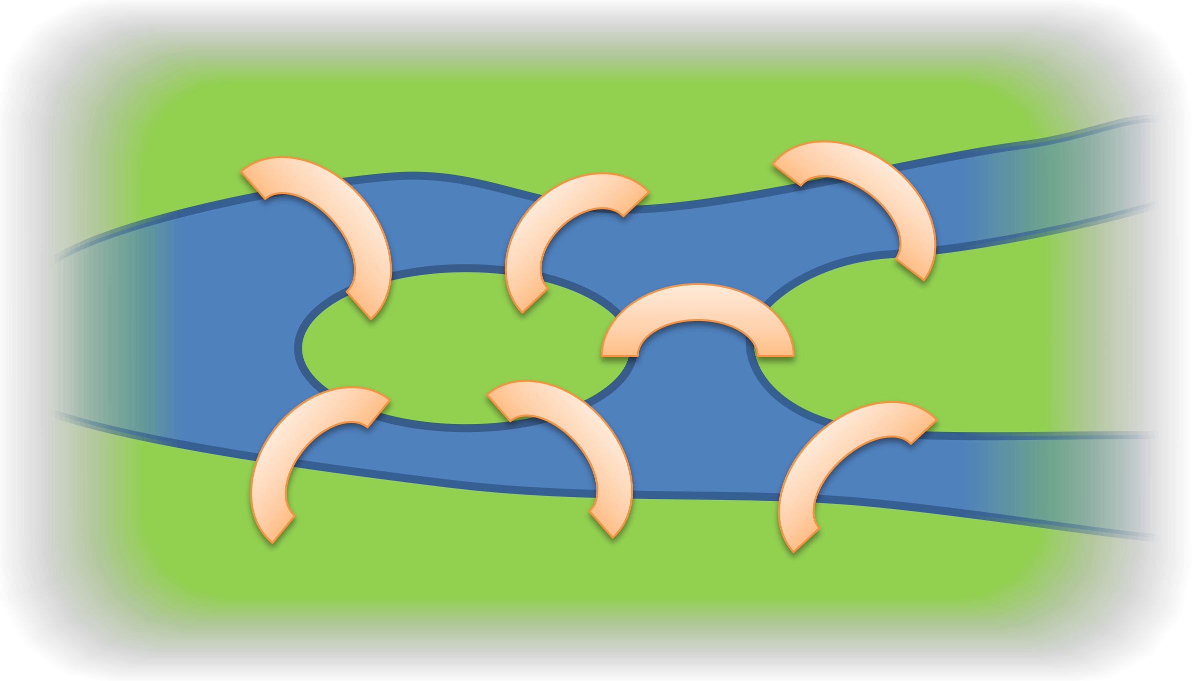
Königsberg

- Used to be Königsberg, Prussia
- Now called Kaliningrad, Russia
- On the Pregel River, including two large islands



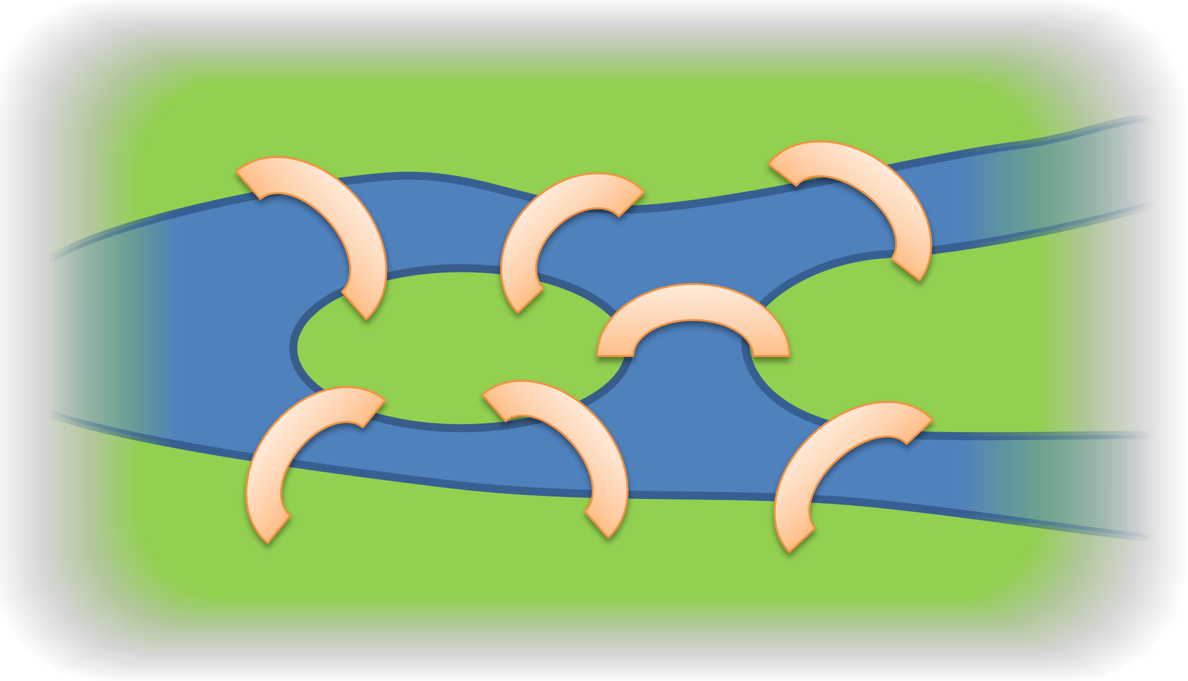
Seven Bridges of Königsberg

- In 1736, the islands were connected by seven bridges
- In modern times, there are only five



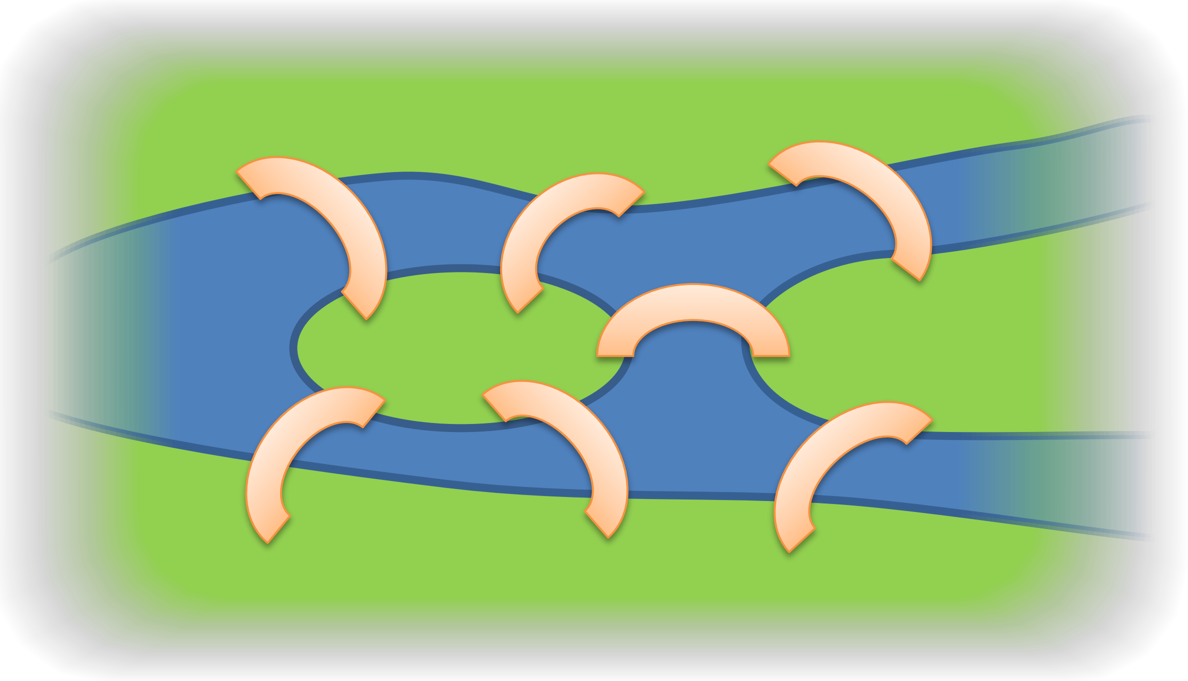
The Challenge

- After a lazy Sunday and a bit of drinking, the citizens would challenge each other to walk around the city and try to find a path which crossed each bridge exactly once



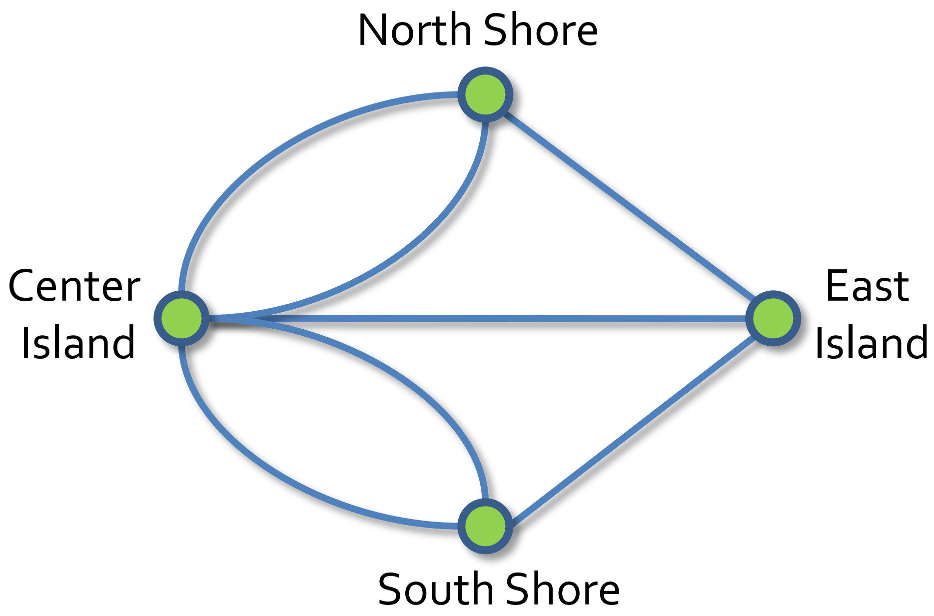
Can you find a solution?

- Can you find such a solution?
- Start anywhere and find a path which crosses each bridge exactly once



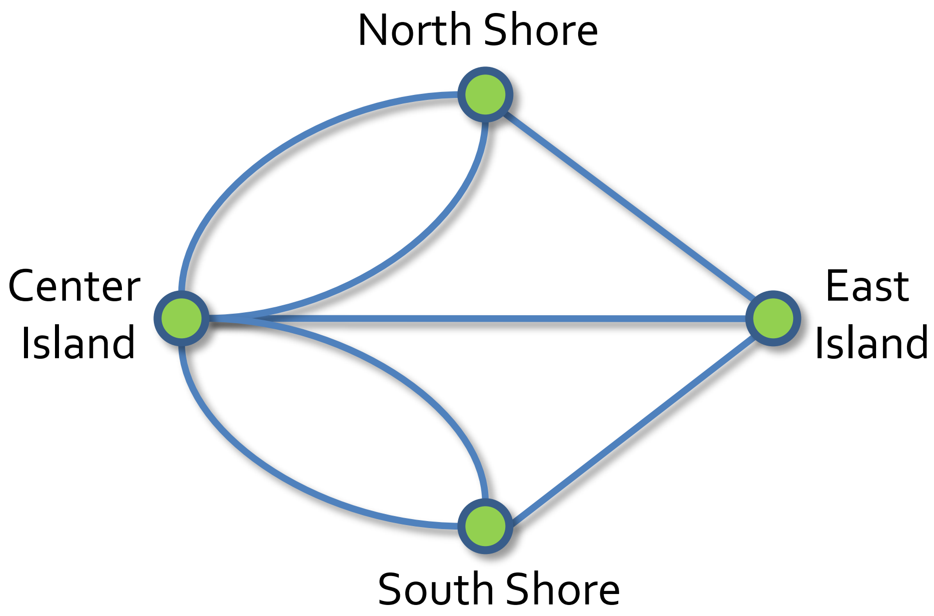
Euler's Solution

- What did Euler find?
- The same thing you did: nothing
- But, he also proved it was impossible
- Here's how:



Graph theoretical view

- By simplifying the problem into a graph, the important features are clear
- To arrive as many times as you leave, the degrees of each node must be even (except for the starting and ending points)

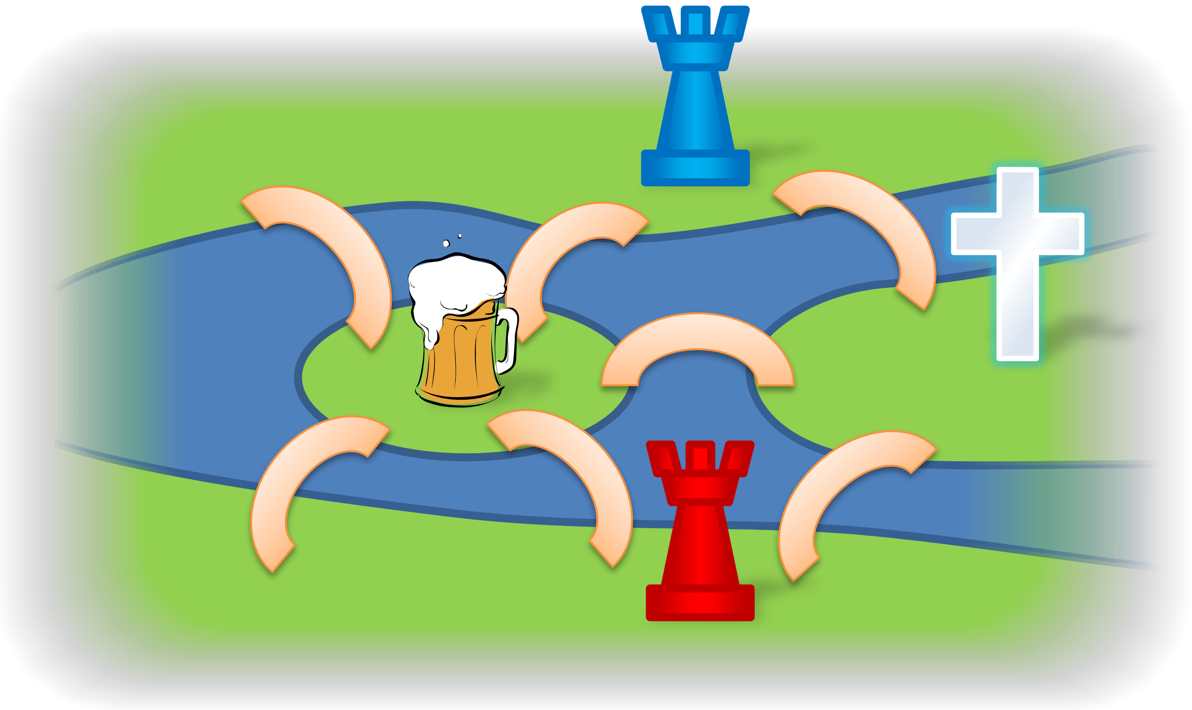


Eulerian Path Algorithm

- There is actually an way to find such a path on a graph where one exists:
  - Start with a node of odd degree if there is one
  - Every time we move across an edge, delete it
  - If you have a choice, always pick an edge whose deletion will not disconnect the graph
- At the end of the algorithm, you will either have an Eulerian path or an Eulerian cycle, depending on the graph

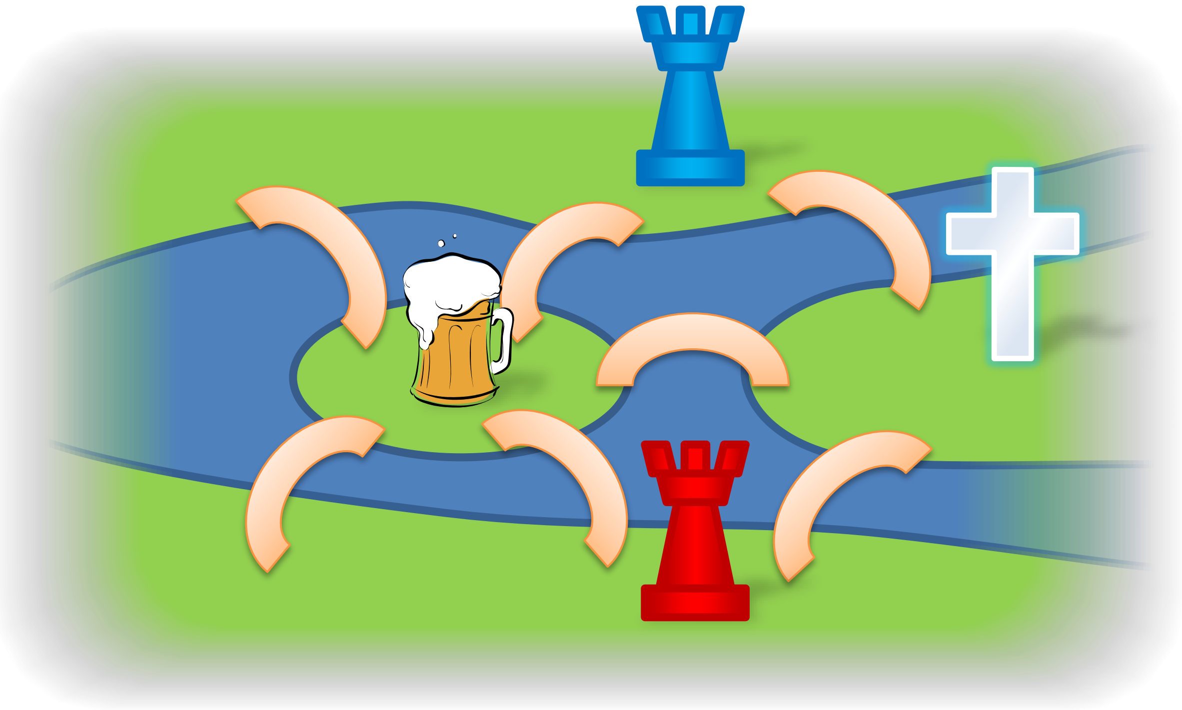
The Two Princes and the Bishop

- We can label the nodes to make a more interesting problem
- Now each piece of land is associated with the Blue Prince, the Red Prince, the Bishop, or the Tavern



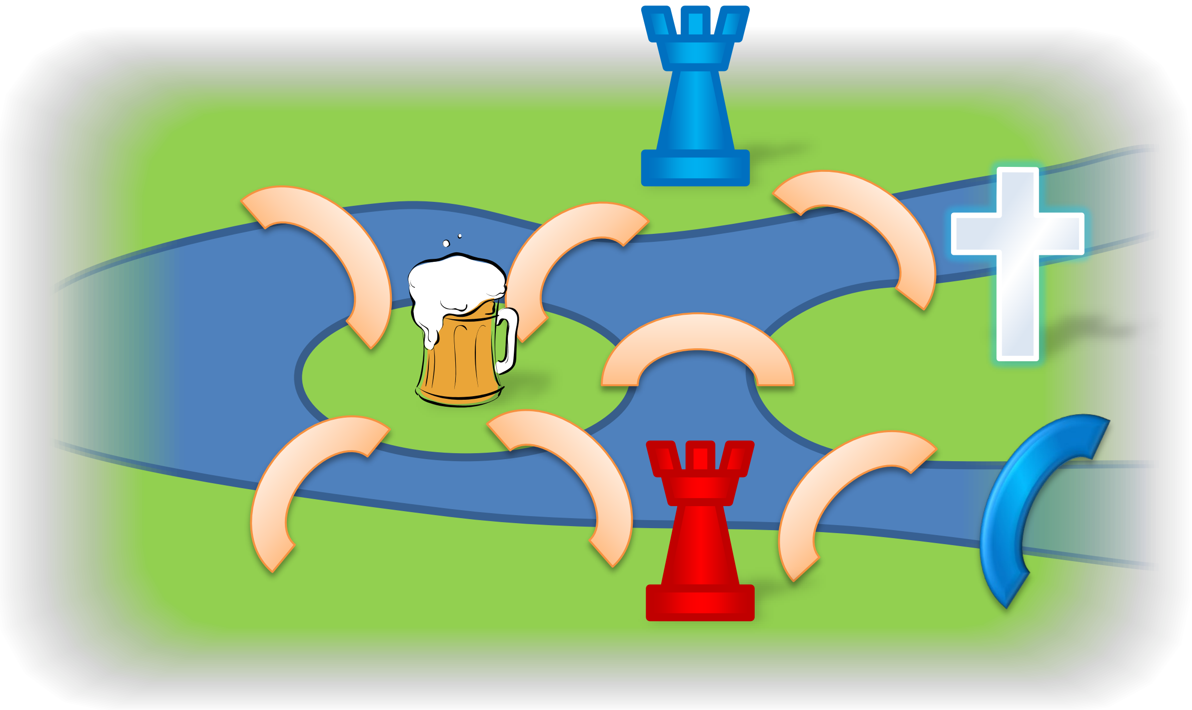
The Blue Prince: 8th Bridge

- The Blue Prince wants to build an 8th bridge so that he can walk starting at his castle, cross every bridge once, and end at the Tavern to brag



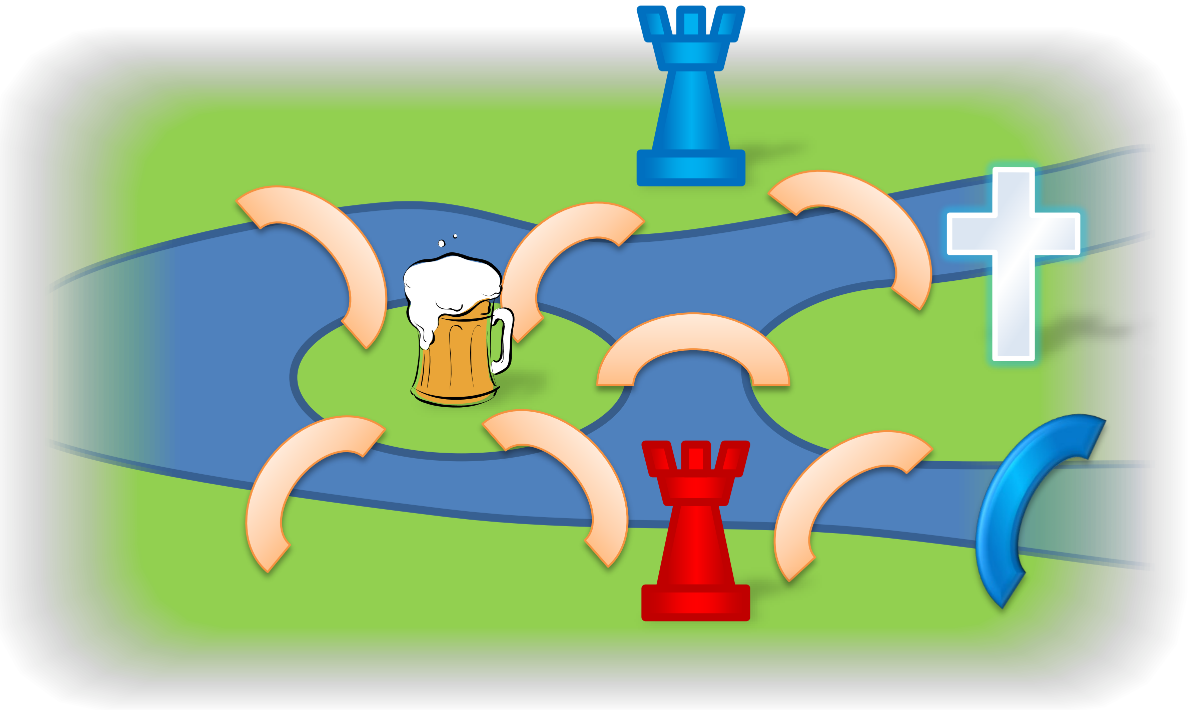
The Blue Prince: Solution

- Put the bridge from the Bishop's land to the Red Prince's land



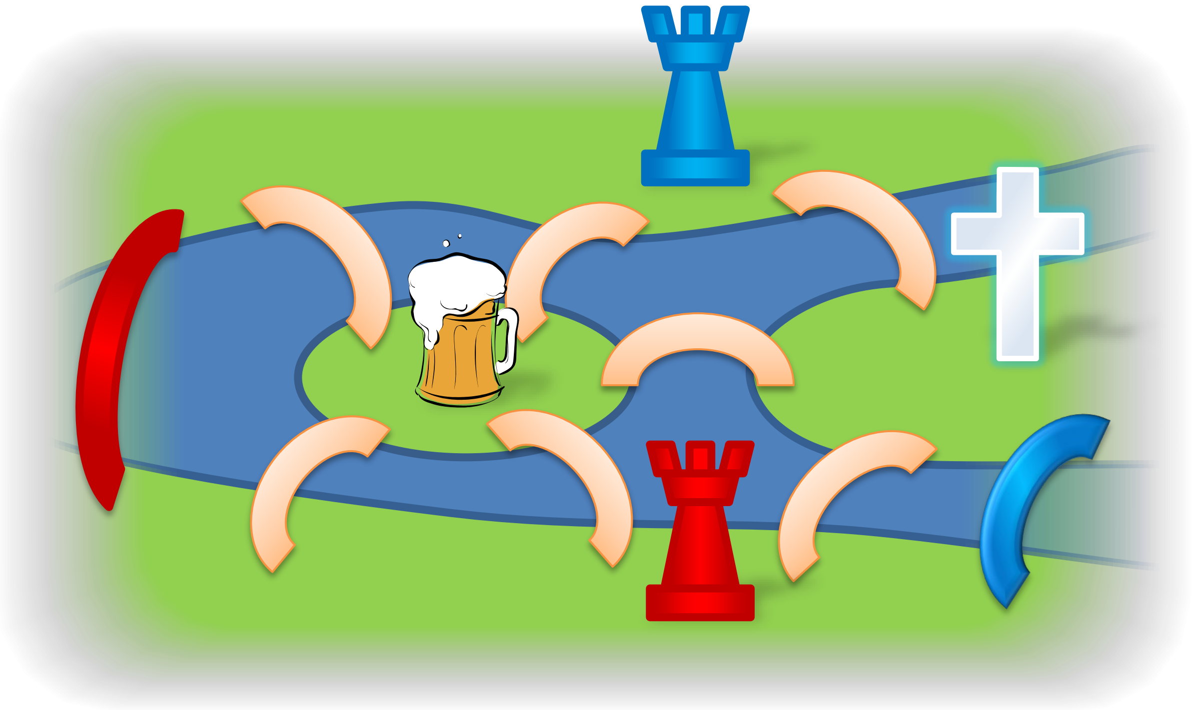
The Red Prince: 9th Bridge

- Furious, the Red Prince wants to build his own bridge so that only he can start at his own castle, cross all the bridges once and then end at the Tavern



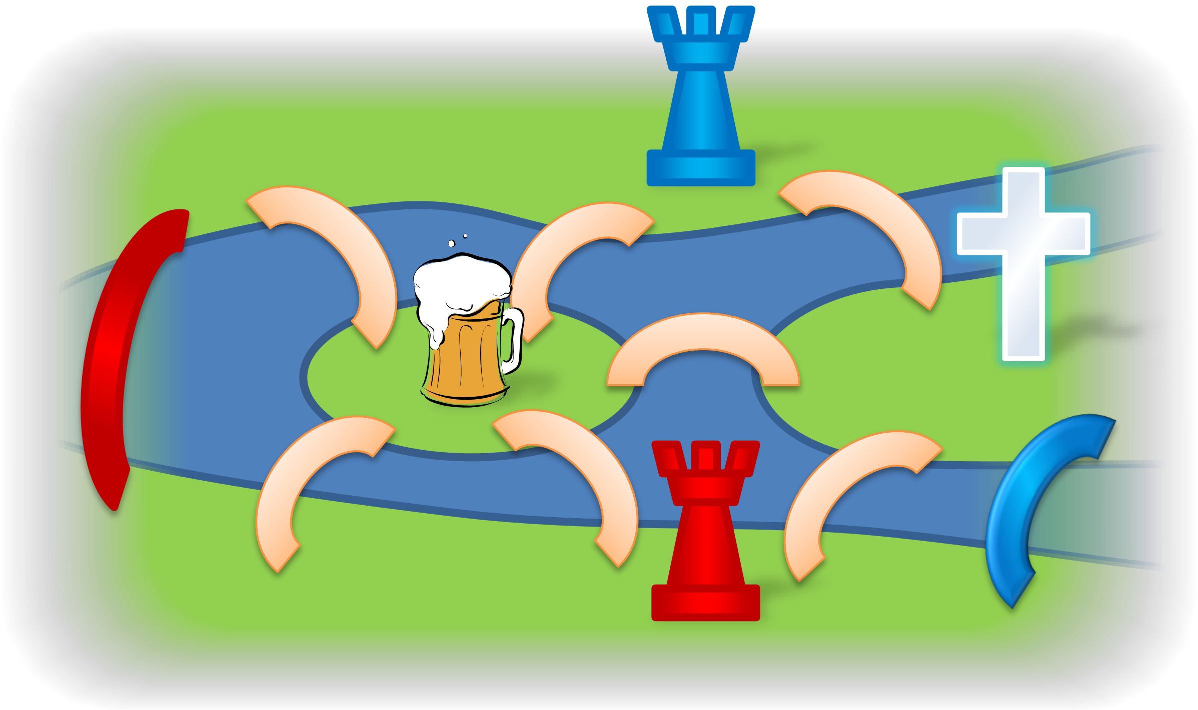
The Red Prince: Solution

- Put the bridge from the Red Prince's land to the Blue Prince's land



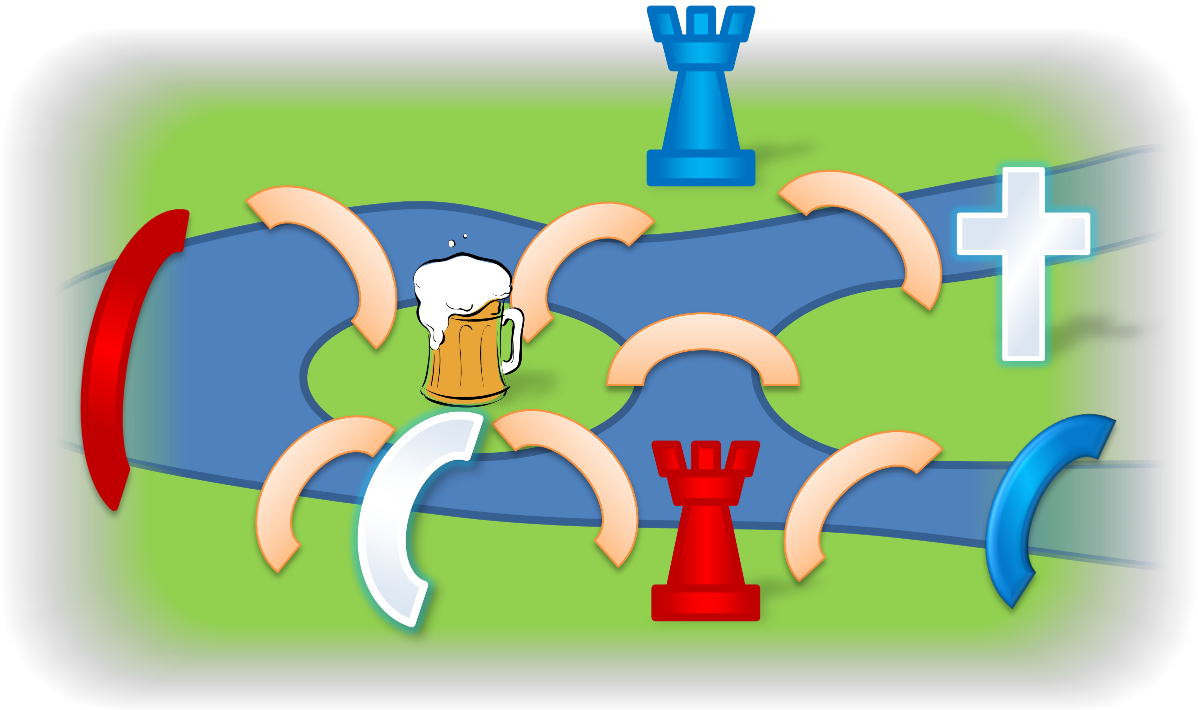
The Bishop: 10th Bridge

- Upset by this pettiness, the Bishop decides to build a 10th bridge which allows all citizens to cross all bridges and return to their starting point



The Bishop: Solution

- Put the bridge from the Center Island to the Red Prince's land, making all pieces of land have even degree



Quiz

Upcoming

Next time…

- More Euler practice
- Network flow
- B-trees

Reminders

- Keep working on Project 3
- Finish Assignment 5
- Read sections 6.2 and 6.4